by Eq. (10),

$$Q_1 = 1/\sigma^2\begin{bmatrix} 1/a & -1/a^2 \\ -1/a^2 & 2/a^3 \end{bmatrix}$$

and using Eq. (11),

$$P_1 = \sigma^2\begin{bmatrix} 2a & a^2 \\ a^2 & a^3 \end{bmatrix}$$

The system transition matrix is

$$\Phi(t) = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}$$

The filter transition matrix is

$$\Phi^*(t) = e^{-at}\begin{bmatrix} 1 - at & t \\ -a^2t & 1 + at \end{bmatrix}$$

and the filter transfer function from Eq. (16) is

$$G(s) = \begin{bmatrix} \dfrac{2as + a^2}{(s+a)^2} \\ \dfrac{a^2s}{(s+a)^2} \end{bmatrix}$$

It is satisfying that the transfer function $G(s)$ provides, as expected, zero steady-state error in both the value and derivative when the measurement is a linear function of time.

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New Information on the Two-Dimensional Jet Interaction Problem

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Introduction

THE injection of gaseous jets perpendicular to supersonic air streams is of interest for supersonic combustion and external burning applications and for jet interaction control devices. The injection process causes separation of the boundary layer upstream of the jet. The resulting recirculation region is responsible for large side forces useful for control devices. This recirculating, nearly stagnant region may control the ignition process when transverse fuel injection is used in supersonic combustion applications.

The two-dimensional flowfield considered in this work results from the injection of a highly underexpanded jet from a converging slot nozzle perpendicular to a supersonic air stream. The boundary layer on the flat surface upstream of the nozzle is turbulent. Pressure and side force data are plentiful from experiments in which air jets have been injected into supersonic air streams.[1–5] However, interaction pressure data for the two-dimensional transverse injection of other gases is meager.[1,6] Data on the effects of changes in jet molecular weight, specific heat ratio, and density are needed if the analyses[1,2] are to be verified. Heat and mass transfer measurements in the upstream recirculation region are not available in the literature. Gas concentrations and residence times in the upstream recirculation region are needed for ignition in the supersonic combustion flowfield to be predicted.

An investigation of the two-dimensional, transverse injection flowfield has been carried out preliminary to supersonic combustion experiments in this flowfield. Hydrogen, helium and nitrogen have been injected perpendicular to a supersonic stream. Wall static pressure, concentration, and temperature measurements have been made upstream of these jets.

Experimental Apparatus

This experiment was conducted in a 4 in. × 6 in. Mach 2.5 blowdown wind tunnel.[7] Before entering the wind-tunnel settling chamber, air passes through a pebble bed heater which, for this series of runs, was maintained at ambient temperature. Both wind-tunnel air and injectant total temperatures remained constant to within a few degrees Rankine during any run. Wind tunnel total temperature and pressure were approximately 525°R and 42 psia, with a resulting Reynolds number of 8×10^6/ft.

The flat plate wind-tunnel model spanned the 4-in. dimension of the wind tunnel. A 0.0080-in.-wide slot nozzle spanned the center 2.00 in. of the model, 5.00 in. downstream of the leading edge. Side plates which extended 1.75 in. upstream of the nozzle contained the entire jet interaction region. The boundary layer was tripped with three dimensional roughness elements 1.00 in. from the leading edge. The boundary layer at separation was always turbulent and approximately 0.10 in. thick. Twenty static pressure taps were located in the plate surface upstream of the jet, with seventeen located on the centerline. Static pressures at taps upstream and downstream of the nozzle were displayed on a 50-tube manometer board and recorded photographically. Five chromel-alumel thermocouples were spot welded to the underside of the plate 0.20 in. from the centerline at locations 0.26, 0.56, 0.81, 1.06 and 2.06 in. upstream of the slot nozzle.

Nitrogen, helium, and hydrogen were injected perpendicular to the air stream from the converging slot nozzle. The jet stagnation pressure to freestream static pressure ratio, p_{0j}/p_f, was varied from 20 to 110. Flow rates were continuously monitored using turbine flowmeters and recorded with jet temperature and pressure so that the nozzle discharge coefficient could be calculated.

Gas samples were taken through the static pressure taps in the recirculation region upstream of the jet. Samples were usually taken from the unmodified taps, but small probes were bonded into the taps during several runs. Eight samples were taken during each wind-tunnel run, usually from three tap locations. Gas samples were stored, and analyzed using a gas chromatograph between wind-tunnel runs.

Results

Wall static pressures, temperatures, and sample composition were measured as the jet strength was varied with each injectant. The ratio of wall static pressure p to freestream static pressure p_f is plotted against the ratio of the distance upstream of the jet x to the slot width d in Fig. 1 for nitrogen injection. As reported by all previous investigators,[1,8] the wall pressure in the separated region increased as the jet strength increased. The upstream pressure distribution

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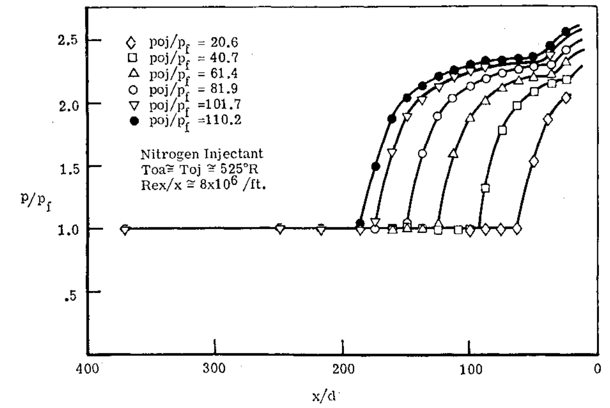

Fig. 1 Variation in wall pressure with jet strength.

was integrated over the interaction area to determine the interaction force F_i. The amplification factor A_c was calculated from the measured interaction force and mass flow rate m_j using the relation

$$A_c = (F_i + F_j)/F_{jv} =$$
$$[F_i + m_j c_e + A_e(p_e - pf)]/(m_j c_e + A_e p_e)$$

where F_j and F_{jv} are the jet thrusts into the freestream static pressure and into vacuum, respectively. The sonic velocity c_e and the jet pressure p_e were calculated at the nozzle exit assuming isentropic flow. A_e is the nozzle exit area. The variation in amplification factor with jet strength is shown for the three injectants in Fig. 2. Although the magnitudes differ, the dependence of amplification factor on jet strength for nitrogen and hydrogen injectants is almost identical to that reported by Werle et al.[5] for injection of air into a Mach 4 freestream.

The difference in the amplification factor between equal strength nitrogen, helium, and hydrogen jets is substantial, as seen in Fig. 2. The wall static pressure distribution upstream of nitrogen, helium, and hydrogen jets is shown in Fig. 3. The jet and freestream temperatures and pressures were identical in these three cases. A systematic variation in the separation distance with the jet molecular weight is observed. Spaid and Zukoski[1] previously observed a 10% increase in separation distance when helium was substituted for nitrogen as the injectant. They concluded, using a simi-empirical theory, that this difference was due to the difference in the specific heat ratio of helium and nitrogen. The fact that hydrogen, with the same specific heat ratio as nitrogen, causes an even greater upstream separation than helium makes this conclusion doubtful. Although the specific heat ratio may be of some importance, the jet molecular weight seems to have a substantially greater effect.

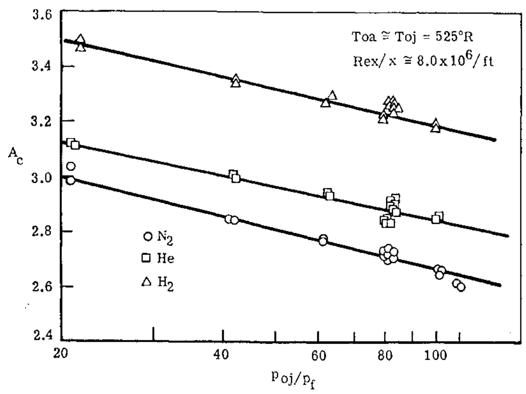

Fig. 2 Variation in amplification factor with jet strength for N_2, He, and H_2 jets.

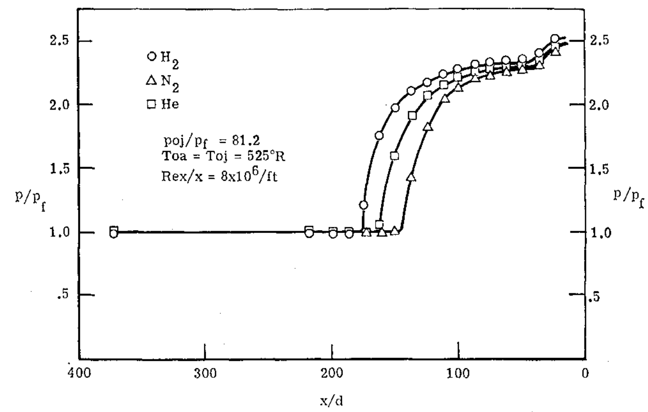

Fig. 3 Wall pressures for equal strength N_2, He, and H_2 jets.

Injectant concentration measurements at the plate surface upstream of the jet, shown in Fig. 4, indicate substantial mixing from the jet into the recirculation region. In this figure, the mole fraction of injectant in the recirculation region X_{inj} is plotted against the ratio of the distance upstream of the jet at which the sample was taken to the distance to the beginning of the separation pressure rise x_{ss}. The injectant concentrations were much higher than expected. Gas samples taken through probes bonded into the static pressure taps indicate that injectant concentrations measured at the wall are almost identical to those measured in the half of the recirculation region closest to the plate surface. The hydrogen concentrations measured indicate that, if similar conditions exist at high temperatures, the upstream recirculation region will be fuel rich. Since velocities in the region are subsonic, static temperatures in the region will be near stagnation conditions, and spontaneous ignition will very likely occur first in this region. Steady-state wall temperatures agree very well with those calculated from the injectant and air concentrations. Concentration measurements in this region have not previously been reported in the literature, so a comparison with previous results cannot be made.

Conclusions

On the basis of this experimental investigation, it is concluded that

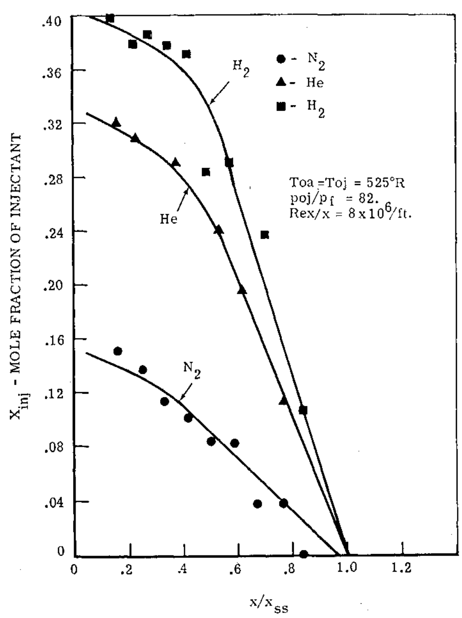

Fig. 4 Injectant concentration in the upstream recirculation region for equal strength N_2, He, and H_2 jets.

1) The two-dimensional jet interaction flowfield depends markedly on the injectant molecular weight. Semiempirical theories that indicate no dependence of the amplification factor on the injectant gas[1] should be modified to consider the flowfield more realistically. Further investigation is necessary to isolate the dominant parameters controlling this complex flowfield.

2) High concentrations of injectant are present in the recirculation region upstream of a two-dimensional transverse jet. Diffusion into this region from the jet will affect the heat-transfer maxima in this region, especially for low-molecular-weight injectants. Combustion will probably be initiated in the upstream recirculation region, when chemically reactive gases are injected perpendicular to a high-enthalpy supersonic stream.

Further work is being conducted into the mechanisms controlling the two-dimensional jet interaction problem, and ignition and combustion in this flowfield.

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